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Gain Scheduling of Observer-Based Controllers with Integral Action

Klaus Trangbaek, Jakob Stoustrup and Jan Dimon Bendtsen

Abstract

This paper presents a method for continuous gain scheduling of observer-based controllers with integral action. Given two stabilising controllers for a given system, explicit state space formulae are presented, allowing to change gradually from one controller to the other while preserving stability. As opposed to previous results, the method allows for non-square systems.

I. INTRODUCTION

Gain scheduling control involves feedback control design based on linear techniques, where the gains of the control law are non-constant. Traditionally, gain scheduling techniques have utilised either simply switching between fixed linear controllers according to some threshold scheme or various forms of linear interpolation between the control signals generated by the individual controllers.

Gain scheduling control is most often applied to control of nonlinear systems that permit linearisation in some operating range or systems with complicated or variable performance criteria. The appeal of employing linear design techniques to deal with such systems is obvious, as a vast selection of tools is available to the control engineer. However, it is not easy to provide stability guarantees for such constructions. For instance, it is possible to find examples of linear systems that are stabilised by two controllers but are in fact unstable for certain linear combinations of the controllers. That is, even if a controller K_0 stabilises a system G and another controller K_1 stabilises the same system, there is no guarantee that $\alpha K_0 + (1 - \alpha)K_1$ stabilises G for all values of $\alpha \in [0, 1]$. Consequently, more advanced scheduling techniques have received considerable attention in recent years [1], [2], [3], [4], [5], [6].

The *Youla-Kucera* parameterisation of all stabilising controllers for a given system has been used for a variety of purposes such as controller switching [7], closed-loop system identification [8], [9], and adaptive controller reconfiguration [10]. See [11] for a survey.

This paper builds on results in [12], [13], where the Youla-Kucera parameterisation was employed to achieve a smooth gain scheduling between observer-based controllers with integral action with certain stability guarantees. In particular, we provide *explicit state space formulae* for a controller construction that achieves stable scheduling between two observer-based controllers with arbitrary numbers of integrators (less than or equal to the number of actuators). Such situations are often encountered in practical applications, where two or more actuators are used to control the same output, for instance due to different dynamical capabilities and/or operating ranges.

After reviewing previous results on observer-based controllers and Youla-Kucera parameterisation, the main result on non-square system is presented in Section V. The method is illustrated by a simulation example in Section VI.

Note that, although gain scheduling is often employed on time varying systems, the theoretical stability guarantees in this paper are only valid for LTI systems, i.e. we only discuss the situation where the plant behaviour does not vary, but we wish to transfer between different controllers. Due to the limited space available we refer the reader to [12], [13], where application to time varying systems is discussed, and where it is also shown how to use the method to achieve bumpless transfer between a large set of controllers.

Notation: The notation is mostly standard. We use plain capital letters for systems described by standard transfer functions

$$H = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

which should be understood as a system mapping an input signal $u \in \mathbb{R}^{n_u}$ to an output signal $y \in \mathbb{R}^{n_y}$ with a state space realisation

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du. \end{aligned}$$

We use calligraphic letters to denote (block) *two-port systems*, e.g.,

$$\mathcal{H} = \left[\begin{array}{c|cc} A & B & B_1 \\ \hline C & D & D_{12} \\ C_1 & D_{21} & D_{22} \end{array} \right]$$

mapping two vector input signals to two vector output signals.

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\star denotes the Redheffer star product [14], i.e. interconnection of systems. Note that if a two-port system is connected with a one-port system, the star product reduces to a linear fractional transformation [14].

\equiv denotes equivalence of transfer functions with no unstable pole-zero cancellations.

$\llbracket A \rrbracket$ denotes the autonomous system $\dot{x} = Ax$.

II. REVIEW OF OBSERVER-BASED CONTROLLERS

This section reviews some of the basic results on observer-based controllers as found in e.g. [14] or [15]. Although trivial, the lemmas here are included to aid the reading of Sections IV and V.

Assuming that the system $y = Gu$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ with

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right] \quad (1)$$

is detectable and stabilisable, an observer-based controller for this system consists of a state observer

$$\dot{\hat{x}} = A\hat{x} + Bu + L_0(C\hat{x} - y) \quad (2)$$

and a state feedback

$$u = F_0\hat{x}. \quad (3)$$

This can be represented by the transfer function $u = K_0y$ with state space representation

$$K_0 = \left[\begin{array}{c|c} A + BF_0 + L_0C & -L_0 \\ \hline F_0 & 0 \end{array} \right]. \quad (4)$$

The classical separation principle states that the observer and the state feedback can be designed separately [15]:

Lemma 1: The system (1) is stabilised by the controller (4) if and only if $A + L_0C$ and $A + BF_0$ are Hurwitz.

Proof: This follows immediately from

$$G \star K_0 = \left[\begin{array}{cc} A & BF_0 \\ -L_0C & A + BF_0 + L_0C \end{array} \right] \equiv \left[\begin{array}{cc} A + BF_0 & -BF_0 \\ 0 & A + L_0C \end{array} \right], \quad (5)$$

where the equivalence follows by a state transformation. \blacksquare

In order to include integral action, the integral of the output can be appended and used in the state feedback. Thus, in addition to (2) we have

$$\dot{x}_I = y = Cx. \quad (6)$$

and apply the control signal $u = F_0\hat{x} + F_{I0}x_I$. The observer is designed as before, i.e. assuring that $A + L_0C$ is Hurwitz,

but the state feedback, $F_e = [F_0 \ F_{I0}]$, is instead designed for extended matrices $A_e = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}$ and $B_e = \begin{bmatrix} B \\ 0 \end{bmatrix}$, so that

$$A_e + B_e F_e = \begin{bmatrix} A + BF_0 & BF_{I0} \\ C & 0 \end{bmatrix}$$

is Hurwitz.

An observer-based controller with integral action then has state space representation

$$K_0 = \left[\begin{array}{cc|c} A + BF_0 + L_0C & BF_{I0} & -L_0 \\ 0 & 0 & I \\ \hline F_0 & F_{I0} & 0 \end{array} \right]. \quad (7)$$

Lemma 2: The system (1) is stabilised by the controller (7) if and only if $A + L_0C$ and $\begin{bmatrix} A + BF_0 & BF_{I0} \\ C & 0 \end{bmatrix}$ are Hurwitz.

Proof: This follows immediately from

$$G \star K_0 = \left[\begin{array}{ccc} A & BF_0 & BF_{I0} \\ -L_0C & A + BF_0 + L_0C & BF_{I0} \\ C & 0 & 0 \end{array} \right] \equiv \left[\begin{array}{ccc} A + BF_0 & BF_{I0} & -BF_0 \\ C & 0 & 0 \\ 0 & 0 & A + L_0C \end{array} \right], \quad (8)$$

where the latter equivalence follows by a state transformation with transformation matrix $\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ I & -I & 0 \end{bmatrix}$. \blacksquare

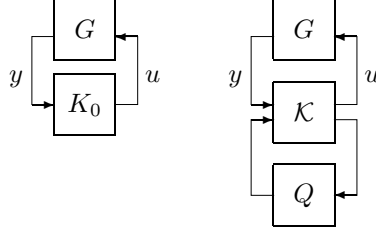


Fig. 1. Left: The interconnection of the system G and the controller K_0 . Right: Controller implemented as $K(Q) = \mathcal{K} \star Q$.

III. YOUNGA-KUCERA PARAMETERISATION

In this section, we provide a brief review of Youla-Kucera parameterisation [16], [17] of stabilising controllers and of the framework established in [6], [13] for gain scheduling control.

Consider the control loop in the left part of Figure 1 and assume that the controller K_0 stabilises the system G . Factorise G as

$$G = NM^{-1} = \tilde{M}^{-1}\tilde{N} \quad (9)$$

with $N, M, \tilde{M}, \tilde{N} \in \mathcal{RH}_\infty$, and K_0 as

$$K_0 = UV^{-1} = \tilde{V}^{-1}\tilde{U} \quad (10)$$

where $U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty$, with the factors chosen to satisfy the double Bezout identity

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U \\ N & V \end{bmatrix} = \begin{bmatrix} M & U \\ N & V \end{bmatrix} \begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

All stabilising controllers for G can now be parameterised according to the Youla-Kucera parameterisation

$$K(Q) = \mathcal{K} \star Q = K_0 + \tilde{V}^{-1}Q(I + V^{-1}NQ)^{-1}V^{-1},$$

with $Q \in \mathcal{RH}_\infty$, i.e., $G \star K(Q)$ is stable for any stable Q and for any stabilising controller K_i , a stable Q exists so that $K(Q) = K_i$. This linear fractional transformation setup is depicted in the right part of Figure 1, and, due to the Bezout identity, can also be implemented as in Figure 2.

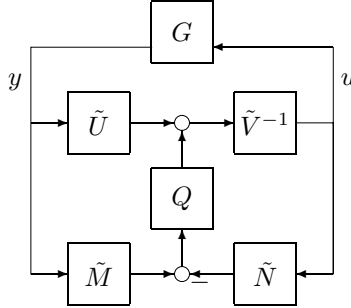


Fig. 2. The controller, $K(Q)$, is composed of coprime factorisations of the controller, K_0 , and the system, G .

Thus, it is possible to implement a given controller as a function of a stable parameter system Q based on another stabilising controller, as depicted in the right part of Figure 1. As stated in [6] this implies that it is possible to change between two controllers online, say, from a nominal controller K_0 to another controller K_1 , in a smooth fashion *without losing stability*, by scaling the Q parameter by a factor $\alpha \in [0; 1]$.

IV. STATE SPACE PARAMETERISATION

In this section we will consider explicit state space formulations of Youla-Kucera parameterisation of observer-based controllers. In [13] the configuration in Figure 3 was used for scheduling a controller gain between two stabilising observer-based controllers with integral action assuming the system to be square, i.e. $n_y = n_u$.

\mathcal{K} is the same as in Figure 1 and Q is then given by $\tilde{K}(\alpha) \star K_1$. The idea behind this setup is that when setting $\alpha = 0$ the resulting controller will be K_0 and when setting $\alpha = 1$ the resulting controller will be K_1 . The loop will be stable for any time varying α , allowing for a smooth transition between the two controllers, simply by changing α slowly.

The following is a modification of a result found in [13].

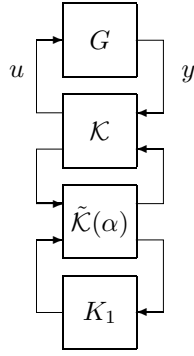


Fig. 3. Gain scheduled control loop.

Lemma 3: Let

$$\bar{K}_0 = \left[\begin{array}{c|c} \bar{A} + \bar{B}\bar{F}_0 + \bar{L}_0\bar{C} & -\bar{L}_0 \\ \hline \bar{F}_0 & 0 \end{array} \right] \quad (11)$$

and

$$\bar{K}_1 = \left[\begin{array}{c|c} \bar{A}_1 + \bar{B}\bar{F}_1 + \bar{L}_1\bar{C} & -\bar{L}_1 \\ \hline \bar{F}_1 & 0 \end{array} \right] \quad (12)$$

be stabilising controllers for the system $\bar{G} = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & 0 \end{bmatrix}$. Then with

$$\mathcal{K} = \left[\begin{array}{cc|cc} \bar{A} + \bar{B}\bar{F}_0 + \bar{L}_0\bar{C} & -\bar{L}_0 & \bar{B} & \\ \hline \bar{F}_0 & 0 & I & \\ \bar{C} & -I & 0 & \end{array} \right], \quad (13)$$

and

$$\tilde{\mathcal{K}}(\alpha) = \left[\begin{array}{cc|cc} \bar{A} & \bar{L}_0 & \bar{B} & \\ \hline -\alpha\bar{F}_0 & 0 & \alpha I & \\ \bar{C} & -I & 0 & \end{array} \right], \quad (14)$$

we have the following properties:

- P1 For $\alpha = 1$, $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star \bar{K}_1 \equiv \bar{K}_1$, i.e. the resulting controller in Figure 3 is \bar{K}_1 .
- P2 For $\alpha = 0$, $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star \bar{K}_1 \equiv \bar{K}_0$, i.e. the resulting controller is \bar{K}_0 .
- P3 The poles of the closed loop system in Figure 3 are identical to the eigenvalues of the matrices

$$\bar{A} + \bar{B}\bar{F}_0, \bar{A} + \bar{L}_0\bar{C}, \text{ and } \begin{bmatrix} \bar{A}_1 + \bar{B}\bar{F}_1 & -\bar{L}_1\bar{C} \\ \bar{A} - \bar{A}_1 & \bar{A} + \bar{L}_1\bar{C} \end{bmatrix},$$

for any α .

Proof: From Lemma 1 we know that $\bar{A} + \bar{L}_0\bar{C}$ and $\bar{A} + \bar{B}\bar{F}_0$ are Hurwitz. The interconnection of \mathcal{K} and $\tilde{\mathcal{K}}(\alpha)$ yields

$$\begin{aligned} \mathcal{K} \star \tilde{\mathcal{K}}(\alpha) &= \left[\begin{array}{cc|cc} \bar{A} + \bar{B}\bar{F}_0 + \bar{L}_0\bar{C} & -\alpha\bar{B}\bar{F}_0 & -\bar{L}_0 & \alpha\bar{B} \\ \hline \bar{L}_0\bar{C} & \bar{A} & -\bar{L}_0 & \bar{B} \\ \bar{F}_0 & -\alpha\bar{F}_0 & 0 & \alpha I \\ -\bar{C} & \bar{C} & I & 0 \end{array} \right] \\ &\equiv \left[\begin{array}{cc|cc} \bar{A} + \bar{B}\bar{F}_0 & (1-\alpha)(\bar{A} + \bar{B}\bar{F}_0) & 0 & (\alpha-1)\bar{B} \\ \hline \bar{L}_0\bar{C} & \bar{A} + \bar{L}_0\bar{C} & -\bar{L}_0 & \bar{B} \\ \bar{F}_0 & (1-\alpha)\bar{F}_0 & 0 & \alpha I \\ -\bar{C} & 0 & I & 0 \end{array} \right], \end{aligned}$$

where the equivalence follows from a state transformation with transformation matrix $\begin{bmatrix} I & -I \\ 0 & I \end{bmatrix}$. Property P1 then follows directly by observing that $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha)$ is an internally stable identity system for $\alpha = 1$.

Computing the entire interconnection in Figure 3 we get

$$\bar{G} \star \mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star \bar{K}_1 = \begin{bmatrix} \bar{A} & \bar{B}\bar{F}_0 & -\alpha\bar{B}\bar{F}_0 & \alpha\bar{B}\bar{F}_1 \\ -\bar{L}_0\bar{C} & \bar{A} + \bar{B}\bar{F}_0 + \bar{L}_0\bar{C} & -\alpha\bar{B}\bar{F}_0 & \alpha\bar{B}\bar{F}_1 \\ -\bar{L}_0\bar{C} & \bar{L}_0\bar{C} & \bar{A} & \bar{B}\bar{F}_1 \\ -\bar{L}_1\bar{C} & \bar{L}_1\bar{C} & -\bar{L}_1\bar{C} & \bar{A}_1 + \bar{B}\bar{F}_1 + \bar{L}_1\bar{C} \end{bmatrix}$$

which through a state transformation with transformation matrix $\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ I & -I & I & -I \\ I & -I & 0 & 0 \end{bmatrix}$ is equivalent to

$$\begin{bmatrix} \bar{A} + \bar{B}\bar{F}_0 & \alpha\bar{B}(\bar{F}_1 - \bar{F}_0) & -\alpha\bar{B}\bar{F}_0 & (\alpha - 1)\bar{B}\bar{F}_0 \\ 0 & \bar{A}_1 + \bar{B}\bar{F}_1 & -\bar{L}_1\bar{C} & 0 \\ 0 & \bar{A} - \bar{A}_1 & \bar{A} + \bar{L}_1\bar{C} & 0 \\ 0 & 0 & 0 & \bar{A} + \bar{L}_0\bar{C} \end{bmatrix}, \quad (15)$$

which proves Property P3.

Property P2 follows from observing that for $\alpha = 0$

$$\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star \bar{K}_1 \equiv \mathcal{K} \star 0 \equiv \bar{K}_0. \quad (16)$$

\mathcal{K} in Lemma 3 provides a standard Youla-Kucera parameterisation [14] with

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \left[\begin{array}{c|cc} \bar{A} + \bar{L}_0\bar{C} & -\bar{B} & \bar{L}_0 \\ \hline \bar{F}_0 & I & 0 \\ \bar{C} & 0 & I \end{array} \right]. \quad (17)$$

Thus, the loop $\bar{G} \star \mathcal{K} \star Q$ is stable for any stable Q . The particular choice $Q = \tilde{\mathcal{K}}(\alpha) \star \bar{K}_1$ provides a method for achieving smooth gain scheduling between two given controllers, while preserving stability.

The last matrix in P3 being Hurwitz is of course equivalent to \bar{K}_1 stabilising G . Note that if $\bar{A}_1 = \bar{A}$, this corresponds to the normal stability conditions for an observer-based controller. We will however need the flexibility of \bar{A}_1 in the following section.

V. STATE SPACE PARAMETERISATION FOR ARBITRARY INTEGRAL GAIN

This section presents the main result of this paper, explicit state space formulae for the configuration in Figure 3 for controllers with integral action. A similar result was presented in [13] but only for square systems, i.e. for $n_y = n_u$. This assumption allowed exploiting a commutative property of the integrators to obtain the parameterisation. However, in case $n_u > n_y$ the result was not valid, since this commutative property does not hold. Here, a result for arbitrary number of integrators will be presented, thus we introduce integral action in the controllers, where the integral gain, F_I , is not necessarily a square matrix.

Theorem 1: Let

$$K_0 = \left[\begin{array}{cc|c} A + BF_0 + L_0C & BF_{I0} & -L_0 \\ 0 & 0 & I \\ \hline F_0 & F_{I0} & 0 \end{array} \right] \quad (18)$$

and

$$K_1 = \left[\begin{array}{cc|c} A + BF_1 + L_1C & BF_{I1} & -L_1 \\ 0 & 0 & I \\ \hline F_1 & F_{I1} & 0 \end{array} \right] \quad (19)$$

be stabilising controllers for the system $G = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ Then with

$$\mathcal{K} = \left[\begin{array}{cc|cc} A + BF_0 + L_0C & BF_{I0} & -L_0 & B \\ 0 & 0 & I & W \\ \hline F_0 & F_{I0} & 0 & I \\ C & 0 & -I & 0 \end{array} \right], \quad (20)$$

and

$$\tilde{\mathcal{K}}(\alpha) = \left[\begin{array}{cc|cc} A & 0 & L_0 & B \\ C - WF_0 & -rI & -I & W \\ \hline -\alpha F_0 & -\alpha F_{I0} & 0 & \alpha I \\ C & 0 & -I & 0 \end{array} \right], \quad (21)$$

where

$$WF_{I0} = rI, \quad r > 0, \quad (22)$$

we have the following properties:

- PI1 For $\alpha = 1$, $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star K_1 \equiv K_1$, i.e. the resulting controller in Figure 3 is K_1 .
- PI2 For $\alpha = 0$, $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star K_1 \equiv K_0$, i.e. the resulting controller is K_0 .
- PI3 The poles of the closed loop system in Figure 3 are identical to the eigenvalues of the matrices

$$\begin{aligned} & \begin{bmatrix} A + BF_0 & BF_{I0} \\ C & 0 \end{bmatrix}, \quad A + L_0 C, \\ & \begin{bmatrix} A + BF_1 & BF_{I1} \\ C & 0 \end{bmatrix}, \quad A + L_1 C, \end{aligned}$$

and $-rI$, for any α .

Proof: Define

$$\bar{A} = \begin{bmatrix} A & 0 \\ C - WF_0 & -rI \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ W \end{bmatrix}, \quad \bar{C} = [C \quad 0]$$

and

$$\begin{aligned} \bar{F}_0 &= [F_0 \quad F_{I0}], \quad \bar{L}_0 = \begin{bmatrix} L_0 \\ -I \end{bmatrix}, \\ \bar{F}_1 &= [F_1 \quad F_{I1}], \quad \bar{L}_1 = \begin{bmatrix} L_1 \\ -I \end{bmatrix}. \end{aligned}$$

Then

$$\left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{C} & 0 \end{array} \right] \equiv \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right] = G \quad (23)$$

and

$$\left[\begin{array}{c|c} \bar{A} + \bar{B}\bar{F}_0 + \bar{L}_0\bar{C} & -\bar{L}_0 \\ \hline \bar{F}_0 & 0 \end{array} \right] = K_0.$$

Furthermore, defining

$$\bar{A}_1 = \begin{bmatrix} A & 0 \\ C - WF_1 & -WF_{I1} \end{bmatrix}$$

we have

$$\left[\begin{array}{c|c} \bar{A}_1 + \bar{B}\bar{F}_1 + \bar{L}_1\bar{C} & -\bar{L}_1 \\ \hline \bar{F}_1 & 0 \end{array} \right] = K_1.$$

Observing that \mathcal{K} and $\tilde{\mathcal{K}}(\alpha)$ are the same as in (13) and (14), Lemma 3 can be applied. Properties PI1 and PI2 are then equivalent to Properties P1 and P2. Computing the matrices in Property P3 we get

$$\begin{aligned} \bar{A} + \bar{B}\bar{F}_0 &= \begin{bmatrix} A + BF_0 & BF_{I0} \\ C & 0 \end{bmatrix} \\ \bar{A} + \bar{L}_0\bar{C} &= \begin{bmatrix} A + L_0C & 0 \\ -WF_0 & -rI \end{bmatrix} \end{aligned}$$

and

$$\begin{bmatrix} \bar{A}_1 + \bar{B}\bar{F}_1 & -\bar{L}_1\bar{C} \\ \bar{A} - \bar{A}_1 & \bar{A} + \bar{L}_1\bar{C} \end{bmatrix} = \begin{bmatrix} A + BF_1 & BF_{I1} & -L_1C & 0 \\ C & 0 & C & 0 \\ 0 & 0 & A + L_1C & 0 \\ W(F_1 - F_0) & -rI + WF_{I1} & -WF_0 & -rI \end{bmatrix}.$$

Property PI3 follows directly from these. ■

Remark 1: In the proof, $-rI$ in Property PI3 has $2n_y$ eigenvalues. However, half of these come from a non-observable part of G as exploited in equation (23). The total order of the controller implementation (i.e. $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star K_1$) is $3(n_s + n_y)$, where n_s is the order of the system G .

Remark 2: \mathcal{K} in Theorem 1 provides a Youla-Kucera parameterisation with

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \left[\begin{array}{cc|cc} A + L_0C & 0 & -B & L_0 \\ -WF_0 & -rI & -W & -I \\ \hline F_0 & F_{I0} & I & 0 \\ C & 0 & 0 & I \end{array} \right]. \quad (24)$$

Remark 3: A W satisfying (22) always exists, since K_0 stabilising G implies that

$$\begin{bmatrix} A + BF_0 & BF_{I0} \\ C & 0 \end{bmatrix}$$

is Hurwitz, which is only possible if F_{I0} has full column rank.

A non-square integral gain F_{I0} may arise even for a square system if integral action is only desired for a subset of the outputs, i.e. we have a number of integrators n_i , which is not equal to n_y . For the case of $n_i \leq n_u$ with no further restrictions on n_y, n_u , we have the following result, which we will state without proof. $S \in \mathbb{R}^{n_i \times n_y}$ is a full row rank matrix selecting the outputs to be integrated.

Theorem 2: Let

$$K_0 = \left[\begin{array}{cc|c} A + BF_0 + L_0C & BF_{I0} & -L_0 \\ 0 & 0 & S \\ \hline F_0 & F_{I0} & 0 \end{array} \right] \quad (25)$$

and

$$K_1 = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & 0 \end{array} \right] \quad (26)$$

be stabilising controllers for the system $G = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$. Then with

$$\mathcal{K} = \left[\begin{array}{cc|cc} A + BF_0 + L_0C & BF_{I0} & -L_0 & B \\ 0 & 0 & S & W \\ \hline F_0 & F_{I0} & 0 & I \\ C & 0 & -I & 0 \end{array} \right], \quad (27)$$

and

$$\tilde{\mathcal{K}}(\alpha) = \left[\begin{array}{cc|cc} A & 0 & L_0 & B \\ SC - WF_0 & -rI & -S & W \\ \hline -\alpha F_0 & -\alpha F_{I0} & 0 & \alpha I \\ C & 0 & -I & 0 \end{array} \right], \quad (28)$$

where

$$WF_{I0} = rI, \quad r > 0, \quad (29)$$

we have the following properties:

- PI4 For $\alpha = 1$, $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star K_1 \equiv K_1$, i.e. the resulting controller in Figure 3 is K_1 .
- PI5 For $\alpha = 0$, $\mathcal{K} \star \tilde{\mathcal{K}}(\alpha) \star K_1 \equiv K_0$, i.e. the resulting controller is K_0 .
- PI6 The poles of the closed loop system in Figure 3 are identical to the eigenvalues of the matrices

$$\begin{bmatrix} A + BF_0 & BF_{I0} \\ SC & 0 \end{bmatrix}, \quad A + L_0C, \\ \begin{bmatrix} A & BC_K \\ B_KC & A_K \end{bmatrix} \quad \text{and } rI$$

for any α .

Remark 4: Although Properties PI3 and PI6 only imply stability for constant α , stability is actually achieved for arbitrarily time varying α due to the block triangular structure of the system (15). If, on the other hand, the plant is time varying, the stability issue becomes much more complex. A discussion of this issue can be found in [13].

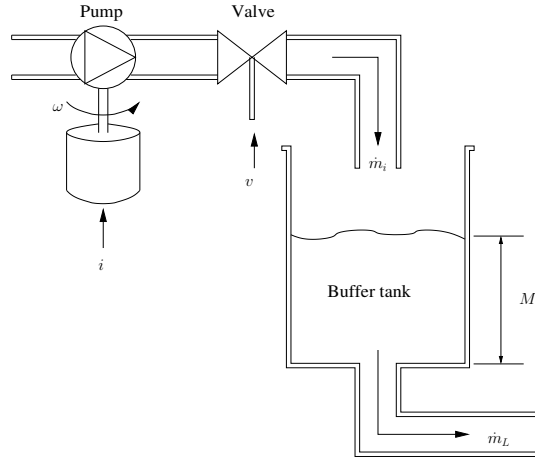


Fig. 4. Buffer tank with inlet flow controlled by pump and valve.

VI. SIMULATION EXAMPLE

To illustrate one use of the result from the previous section we consider the buffer tank in Figure 4. The mass of fluid in the buffer tank, M , is maintained by controlling the inlet flow \dot{m}_i . The outlet flow \dot{m}_L acts as a disturbance. The inlet flow is controlled by two actuators, the pump and the valve. The pump has a high moment of inertia and is therefore slower than the valve. On the other hand, we wish to keep the valve close to fully opened in order to preserve energy. We therefore use the valve to compensate for fast disturbances, and let the pump take care of the steady state compensation. The only measurement is the fluid mass, M , whereas we have two inputs, the current i driving the electrical pump and the valve position, v , which can be controlled directly in the interval $[0, 1]$. Under some simplifying assumptions the system can be modelled as

$$\begin{aligned}\frac{d}{dt}M(t) &= \dot{m}_i(t) - \dot{m}_L(t) \\ \frac{d}{dt}\omega(t) &= \frac{1}{J}(\beta_4 i(t) - f\omega(t)) \\ \dot{m}_i(t) &= \frac{\beta_3 v(t)}{\beta_2 + \beta_1 \beta_3 v(t)}\omega(t),\end{aligned}$$

where ω is the rotational speed of the pump. $J = 0.1 \text{ kgm}^2$ is the moment of inertia, $f = 10^{-4} \frac{\text{Nm s}}{\text{rad}}$ represents friction, and $\beta_1 = 50 \frac{\text{rad}}{\text{kg}}$, $\beta_2 = 50 \frac{\text{rad}}{\text{s bar}}$, $\beta_3 = 1.25 \frac{\text{kg}}{\text{s bar}}$, $\beta_4 = 0.1 \frac{\text{Nm}}{\text{A}}$ are constants representing pump and valve characteristics.

In the operating point $M_0 = 10 \text{ kg}$, $\omega_0 = 1000 \frac{\text{rad}}{\text{s}}$, $\dot{m}_{i0} = \dot{m}_{L0} = 10 \frac{\text{kg}}{\text{s}}$ with control signals $v_0 = 0.8$, $i_0 = 1 \text{ A}$ we have the (unstable) linearised model

$$\begin{aligned}\frac{d}{dt}x &= Ax + Bu + B_d d \\ y &= Cx\end{aligned}$$

where

$$\begin{aligned}A &= \begin{bmatrix} 0 & 0.01 \\ 0 & -0.001 \end{bmatrix}, \quad B = \begin{bmatrix} 6.25 & 0 \\ 0 & 10 \end{bmatrix}, \\ C &= [1 \quad 0], \quad B_d = \begin{bmatrix} -1 \\ 0 \end{bmatrix},\end{aligned}$$

with state and input vectors

$$x(t) = \begin{bmatrix} M(t) - M_0 \\ \omega(t) - \omega_0 \end{bmatrix}, \quad u(t) = \begin{bmatrix} v(t) - v_0 \\ i(t) - i_0 \end{bmatrix},$$

disturbance $d(t) = \dot{m}_L(t) - \dot{m}_{L0}$, and measurement $y(t) = M(t) - M_0$.

We design two separate controllers for this system. K_0 with $F_0 = \begin{bmatrix} -0.33 & -0.0016 \\ -0.026 & -0.0051 \end{bmatrix}$, $F_{I0} = \begin{bmatrix} -0.32 \\ -0.0013 \end{bmatrix}$, and $L_0 = \begin{bmatrix} -1.0945 \\ -9.8911 \end{bmatrix}$ is designed for disturbance rejection relying heavily on valve action for fast responses. K_1 , with

$$F_1 = \begin{bmatrix} -0.0052 & -0.0002 \\ -0.37 & -0.026 \end{bmatrix}, \quad F_{I1} = \begin{bmatrix} -0.0005 \\ -0.0270 \end{bmatrix},$$

and $L_1 = L_0$, does not use the valve as heavily, resulting in a less efficient disturbance rejection. In periods of time where the outlet is expected to be fairly steady, this controller can be used with the steady state valve position closer to the saturation limit, thus minimising power consumption.

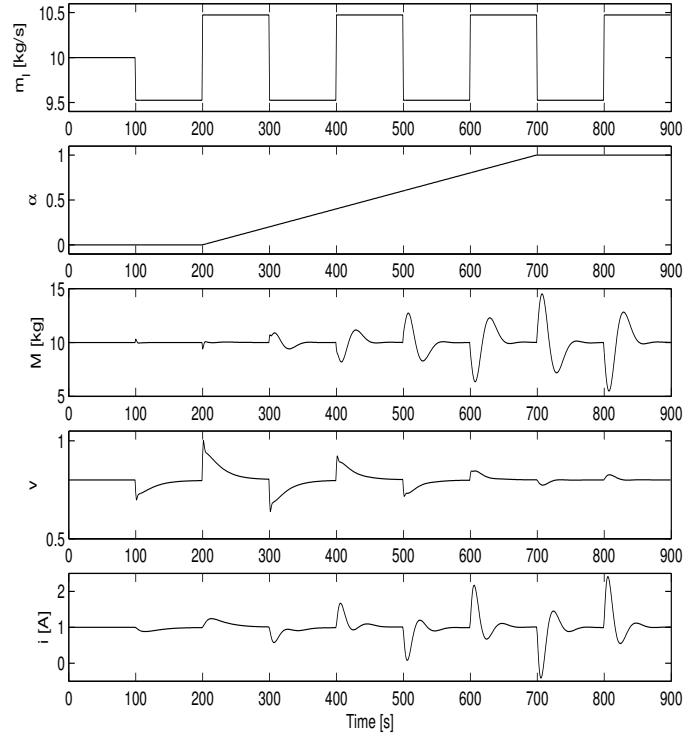


Fig. 5. Simulation example.

Figure 5 shows a simulation using the method presented in the previous section. We assume that for some reason we want to change slowly from K_0 to K_1 . From top to bottom, the plots show the outlet flow \dot{m}_L acting as a disturbance, the scheduling variable α , the fluid level M , the valve position v , and the pump current i . As α is increased, the resulting controller changes continuously from K_0 to K_1 . This results in an expected degradation of the disturbance rejection, but also minimises the valve actuation. During the transition, stability is preserved and the behaviours of the two controllers is mixed in a sensible manner.

One might have considered simply mixing the gains, i.e. implementing a controller with $F_m = (1 - \alpha)F_0 + \alpha F_1$, $F_{Im} = (1 - \alpha)F_{I0} + \alpha F_{I1}$, and $L_m = (1 - \alpha)L_0 + \alpha L_1$, but then no guarantee of stability would be given. In the above example, stability would have been preserved for $\alpha \in [0, 1]$, but for $\alpha = 0.96$, the closed-loop would have poorly damped poles in $s = -0.08 \pm 0.3j$ resulting in very poor performance.

This example demonstrates how the Youla-Kucera parameterisation can be used for a system, where direct gain mixing would lead to problems even when scheduling slowly. Examples of using the method for fast scheduling and for time-varying systems can be found elsewhere, e.g. in [12].

VII. CONCLUSIONS

In this paper a method for continuous gain scheduling of observer-based controllers has been presented. In contrast to previous results, there is no requirement for the system to be square. The method allows continuous scheduling between two given stabilising controllers, preserving stability at all times. The method was illustrated through simulations of a fluid buffer system.

We note that all results can easily be extended from continuous to discrete time, along the lines of [12].

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